DISTORTIONLESS RF PULSE WIDTH MODULATION

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ABSTRACT

A method is presented to synthesize a switching signal which linearly encodes a complex-modulated RF signal to an RF carrier frequency. The switching distortion associated with this method is limited to high-pass components out of band. Consequently, the switching signal may be filtered after high efficiency amplification to produce the linear RF modulation. The method requires a switching frequency slightly higher than the highest frequency in the band of interest.

1. INTRODUCTION

Conventional RF transmitter take baseband analog in-phase and quadrature signals and mix them to an RF frequency. Severe linearity requirements are placed on the subsequent amplification stages which then drive a power amplifier (PA). PA's fall into two categories: linear PA's or switching PA's. Linear PA's display high linearity at the cost of low power efficiency. Classically, switching PA's display high efficiency but only support constant-envelope (phase- or frequency-modulation) formats.

We propose extending these switching PA's to linear modulation by constructing a switching signal which is comprised of the desired linear modulation plus out-of-band distortion. This switching signal can be amplified with high efficiency switching amplifiers and can then be filtered to remove switching distortion, preserving a replica of the desired linear RF modulation. Thus, a transmitter with high linearity and efficiency RF power amplification can be realized. The signalling scheme presented in this paper is termed distortionless RF pulse-width modulation (RF-PWM).

The framework for this signalling scheme is a modulation format called click modulation [1, 2]. However, it has been shown that click modulation is really a pulse-width modulation (PWM) scheme with prefiltering [3]. Consequently, the signalling scheme presented in this paper is a novel form of pulse-width modulation which up-converts a signal from baseband to an RF carrier.

2. BACKGROUND

Methods have been proposed before to represent a modulated RF signal as a switching signal. The use of a 1-bit sigma-delta loop running at $4 \times f_c$ (f_c being the RF carrier frequency) is the simplest choice [4]. However, this signalling scheme has transitions at the rate of $4 \times f_c$; this means that it has twice the number of transitions that a constant-envelope signal would have. This is undesirable since such a signal would create twice the switching loss in a PA. However, the operations required of a 1-bit sigma delta are very simple, and therefore do not present great signal processing hurdles. Another method has been proposed, extending the notion of pulse-width modulation to include phase modulation [5]. Due to the RF nature of the signalling, a trigonometric mapping must be used to compute the desired pulse width. These polar-based mappings are intensive calculations, since the intermediate quantities are not bandlimited. However, the signals switch at a rate of $2 \times f_c$ on average. Due to the non-bandlimited nature of the polar mappings however, higher order harmonics are not bandlimited and therefore interfere with the band of interest. Consequently, this method of RF-PWM is not strictly distortionless and is limited to small percentage bandwidths.

3. CLICK MODULATION

A brief description of click modulation is presented here. For a derivation, see the original paper by B. F. Logan [1]. The basic gist is as follows.

Click modulation generates a pulse which represents a baseband signal (without dc) which is limited to frequencies f which satisfy

$$|f| \in [\mu, \nu]\,. \tag{1}$$

The desired real signal $s(t)$ should satisfy

$$(\mathrm{sgn}\, s(t)) \otimes K(t) = x(t)\,, \tag{2}$$

where $K(t)$ is a low-pass reconstruction filter and $x(t)$ is the input to the system and is the signal which is to be represented as a pulse. The output of the system is

$$y(t) = \mathrm{sgn}(s(t))\,. \tag{3}$$

This may also be written, for a real signal $s(t)$,

$$Im\{\log(s(t))\} \otimes K(t) = x(t)\,, \tag{4}$$

since Im log() is spectrally equivalent to sgn(). One such solution to this equation is to set

$$s_a(t) = e^{-jx(t)}\,. \tag{5}$$

However, this signal is not real; we desire a real-valued signal that obeys (4). Logan presents a theorem which says that if:

1. one produces the signal
$$F(t) = x(t) + j\hat{x}(t)\,, \tag{6}$$
where
$$\hat{x}(t) = x(t) \otimes \frac{1}{\pi t} \tag{7}$$
is the Hilbert transform of $x(t)$, and
2. one does not modify the spectrum of $F(t)$ within $[-\infty, \alpha]$,

then the spectra of any analytic function of $F(t)$ (including $\log F(t)$ and $e^{-jF(t)}$) will be unmodified within $[-\infty, \alpha]$ [6].

So, our goal is to modify $s_a(t)$ to make it real-valued without changing its spectrum within $[-\infty, \alpha]$. We let

$$Z(t) = e^{-jF(t)} \tag{8}$$

and then bandlimit it to β, by setting

$$z(t) = Z(t) \otimes K_{\alpha,\beta}(t), \tag{9}$$

where $K_{\alpha,\beta}(t)$ is a low-pass filter whose frequency response is identically one up to α and is zero past β, $\beta > \alpha > \nu$. Then, $z(t)$ is bandlimited but log $z(t)$ still recreates $x(t)$. Now, if we synthesize the signal

$$G(t) = z(t) + z^*(t)e^{j4\pi ct}, \tag{10}$$

the term $z^*(t)e^{j4\pi ct}$ has spectral support from $[2c-\beta, 2c]$. Therefore, if we let $2c - \beta > \alpha$, this will still recreate $x(t)$. One thing to note is that $G(t)$ can be written as

$$G(t) = e^{j2\pi ct}[z(t)e^{-j2\pi ct} + z^*(t)e^{j2\pi ct}]. \tag{11}$$

The term in the brackets is a real signal $s(t)$. Therefore, $\mathrm{Im}\{\log(G(t))\} = \mathrm{Im}\{\log(s(t))\} + 2\pi ct + 2n_t\pi$. We need the $2n\pi$ since the Im log( ) of a real signal is either 0 or π, however, with the additional phase of $2\pi ct$, the result $\angle s(t) + 2\pi ct$ would be more than π. Therefore, we need an integral multiple of 2π.

The main point to note is that $s(t)$ makes jumps up and down in phase, while $G(t)$ makes jumps only downward, because it is biased with a $2\pi ct$ ramp. Consequently, sgn($s(t)$) (which is spectrally equivalent to $\mathrm{Im}\{\log(s(t))\}$ does not spectrally represent $g(t)$ (nor $x(t)$). However, if $s(t)$ did make phase jumps only upward, then it would represent $G(t)$ (and therefore $x(t)$). To facilitate these positive jumps, Logan explicitly introduces negative jumps in-between the jumps of $s(t)$, therefore flipping the $s(t)$'s negative jumps and making them positive. These explicit negative jumps are periodic with period $1/(2c)$. Therefore, they introduce distortion at multiples of $2c$, which is out of our band of interest. Note that the switching frequency is $2c$--that is, the output $y(t)$ rises and falls once every $1/(2c)$.

PWM produces a linear replica of the duty ratio plus switching harmonics. These switching harmonics include phase-modulated replicas of the duty ratio [3]. Due to the nonlinearity of the phase-modulation process, the switching harmonics are not strictly bandlimited (although they decay quickly) and interfere with the baseband signal.

Click modulation offers a way to bandlimit these harmonics. It effectively pre-filters the phase modulated signals such that they don't interfere with the baseband signal. However, one must do so such that the baseband signal is also not perturbed. By performing Hilbert transforms and converting to analytic signals, Logan is able to pre-filter the duty ratio such that it is not perturbed within the band of interest. Yet, upon phase-modulation, it is still bandlimited.

In most PWM systems, the switching frequency can be chosen to be large enough such that the phase-modulated replicas of the duty ratio have decayed enough to be negligible within the band of interest. However, in the case of RF-PWM minimizing the switching frequency is critical. Consequently, we seek a method of producing our RF signal with low distortion and with a low switching frequency.

4. DERIVATION

Distortionless RF-PWM attempts to produce a signal

$$y(t) = \mathrm{sgn}(s(t)). \tag{12}$$

With the property that

$$y(t) \otimes K(t) = x(t), \tag{13}$$

where $x(t)$ is given by

$$x(t) = \mathrm{Re}\{u(t)e^{j2\pi f_c t}\}, \tag{14}$$

and $u(t)$ is a complex-valued signal bandlimited to $[-B, B]$. $K(t)$ is a band limiting reconstructive filter. $K(t)$ is ideally a brick-wall filter covering a band of width $2B$ centered at f_c. Its Fourier transform $K_F(f)$ is given by

$$K_F(f) = \begin{cases} 1, f_c - B < |f| < f_c + B \\ 0, \text{ elsewhere} \end{cases} \tag{15}$$

Equations (13) and (15) define the term distortionless. It means that any distortion components of the signal $y(t)$ fall out of the band of interest—that is, out of the support of $K_F(f)$.

From the theory of click modulation, one would compute the Hilbert transform, take the exponential, low-pass filter, and then produce the square wave representation. However, due to the band-pass nature of the signal, all these steps are not necessary. Consequently, we simplify the necessary computations greatly. First, the Hilbert transform of $x(t)$ is given by

$$\hat{x}(t) = \mathrm{Im}\{u(t)e^{j2\pi f_c t}\}, \tag{16}$$

which requires minimal computation. Therefore,

$$F(t) = u(t)e^{j2\pi f_c t}. \tag{17}$$

Next, we take the exponent and expand it as a Taylor series

$$Z(t) = e^{-jF(t)} = 1 + \sum_{n=1}^{\infty} \frac{(-ju(t))^n}{n!} e^{jn2\pi f_c t}. \tag{18}$$

However, since we will next low-pass filter, we may choose our filter $K_{\alpha,\beta}$ to have $\alpha = \beta = f_c + B$. Provided that $f_c > 2B$, each of the terms in Equation (18) is spectrally orthogonal and we may choose only the terms which have frequency content below α. Therefore,

$$Z(t) \otimes K_{\alpha,\beta} = 1 - ju(t)e^{j2\pi f_c t}. \tag{19}$$

This simplification represents an immense reduction in complexity, since no exponential needs to be evaluated (therefore omitting the need for a look-up table). In addition, from Logan's theory, the filter $K_{\alpha,\beta}$ must have frequency response identically one up to α, and be identically zero after β. These constraints are highly impractical if one were to actually design the filter, especially if one wants to make the transition band, β-α very small. In effect, they would result in an extremely high-order filter, which would therefore involve much computation. Since we can set $\alpha = \beta$, we do not lose any spectral efficiency in leaving space for a transition band. This allows us to use the minimum switching frequency possible by letting $c=(\alpha+\beta)/2$. This system is shown in Figure 1 as click modulation, and in Figure 2 as NSPWM.

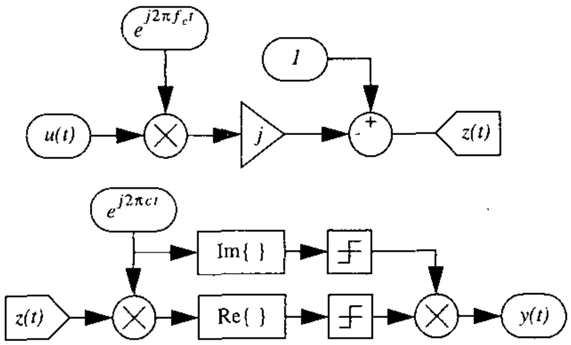

Figure 1. Single-Sided Distortionless RF PWM implemented as Click Modulation

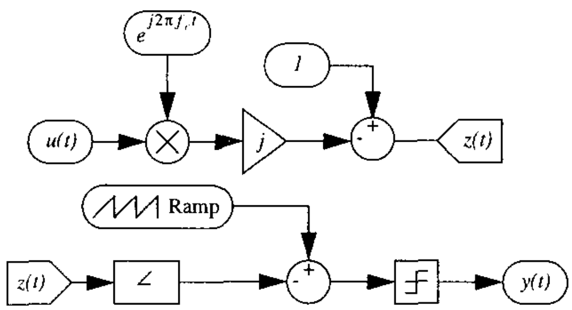

Figure 2. Single-Sided Distortionless RF PWM implemented as PWM

One shortcoming with the above system is that it must switch at $2c$, which equals $2fc+2B$. However, the system is currently modulating only one pulse edge; the other edge is periodic. It would be nice if one could halve the switching frequency and modulate both edges. This is equivalent to making the phase jumps of $G(t)$ alternate positive and negative. One method of achieving the desired result would be to make every other phase jump of $G(t)$ be a negative jump. The transition time would need to be changed, of course. If one were to input $-u(t)$ into the system, one would obtain $-y(t)$ at the output. Correspondingly, one should be able to alternate jumps of $G(t)$ formed by putting $u(t)$ and $-u(t)$ into the system. By inverting every other jump, one would obtain the desired result. To derive the necessary system, assume that $G_1(t)$ corresponds to $u(t)$ and $G_2(t)$ corresponds to $-u(t)$. Then,

$$\begin{aligned} G_1(t) &= z_1(t) + z_1{}^*(t)e^{j4\pi ct} \\ &= (1 - ju(t)e^{j2\pi f_c t}) + (1 + ju(t)e^{j2\pi f_c t})e^{j2\pi ct} \end{aligned} \quad (20)$$

and

$$\begin{aligned} G_{2a}(t) &= z_{2a}(t) + z_{2a}{}^*(t)e^{j4\pi ct} \\ &= (1 + ju(t)e^{j2\pi f_c t}) + (1 - ju(t)e^{j2\pi f_c t})e^{j2\pi ct} \end{aligned} \quad (21)$$

However, since $G_1(t)$ and $G_{2a}(t)$ are modulated with a positive phase, their jumps are both in a negative direction; we want them to be in the opposite directions. Therefore, we set

$$G_2(t) = G_{2a}{}^*(t) = z_2{}^*(t) + z_2(t)e^{-j4\pi ct}. \quad (22)$$

Now, we find that $\mathrm{Im}\{\log(G_1(t))\}$ makes negative transitions twice every $1/c$, and $\mathrm{Im}\{\log(G_2(t))\}$ makes positive transitions twice every $1/c$. We wish to multiplex these transitions, taking one negative transition from $\mathrm{Im}\{\log(G_1(t))\}$ and one from $\mathrm{Im}\{\log(G_2(t))\}$, giving one negative and one positive transition every $1/c$. This is equivalent to reducing the modulation by one half so that $\mathrm{Im}\{\log(G_1(t))\}$ makes a negative transition once every $1/c$ and $\mathrm{Im}\{\log(G_2(t))\}$ makes a positive transition once every $1/c$:

$$G_1(t) = z_1(t) + z_1{}^*(t)e^{j2\pi ct} \quad (23)$$

$$G_2(t) = z_2{}^*(t) + z_2(t)e^{-j2\pi ct} \quad (24)$$

However, we are not guaranteed that the transitions will be interspersed—for example, G_1 could make a transition, G_2 could make two transitions, and then G_1 would make its transition. The output would go negative, positive, negative (due to G_2's second transition), and then positive again. This is catastrophic, since G_1 should only modulate a negative transition and G_2 should only modulate a positive transition. In fact, we are almost guaranteed that this double-transition will occur since F has frequency content close to c and therefore flips phase approximately every $1/(2c)$. This situation is shown in Figure 3.

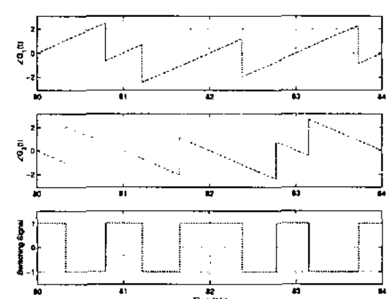

Figure 3. Illustration of Double-Transition in $G_1(t)$ and $G_2(t)$

To combat this double-switching condition, we offset G_2's phase ramp by $\pi/2$. Therefore, we finally have

$$G_2(t) = -jz_2{}^*(t) + jz_2(t)e^{-j2\pi ct}, \quad (25)$$

and c is the switching rate. The signals of interest are shown in Figure 4.

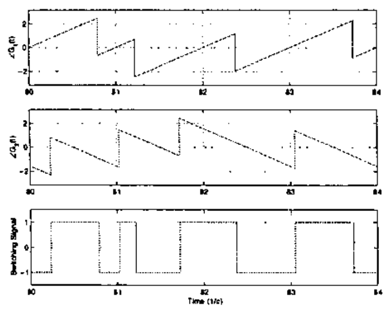

Figure 4. Illustration of Signals With $G_2(t)$ Offset in Phase

Note that although the signal switches on average at a rate c, there are periods where there are two positive transitions; these periods are compensated by a period with no positive transition.

The resulting system switches at a rate c. However, c still equals $(\alpha+\beta)/2$. Recall that previously, we set $\alpha=\beta=f_c+B$. Therefore, the system can switch at a rate f_c+B. The final system is shown in Figure 5. Spectral plots of the output are shown in Figure 6 and in Figure 7, for a carrier frequency f_c of 1.024 GHz, an input of two complex tones at 22 MHz and 32 MHz, and a switching frequency of $1.4f_c$. The output is distortionless up to the switching frequency.

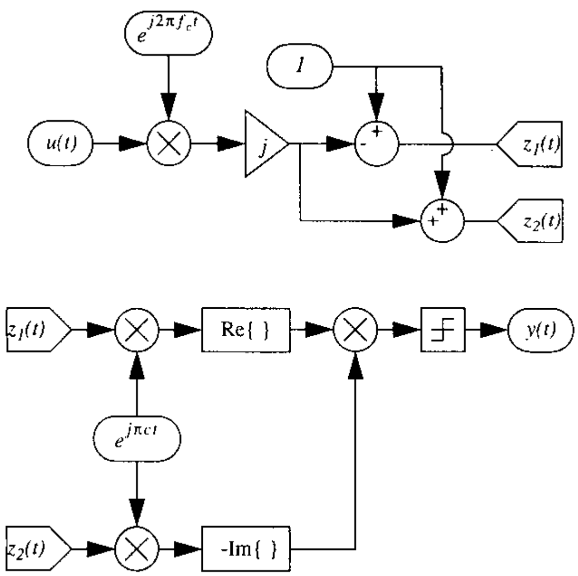

Figure 5. Two-Sided Distortionless RF PWM

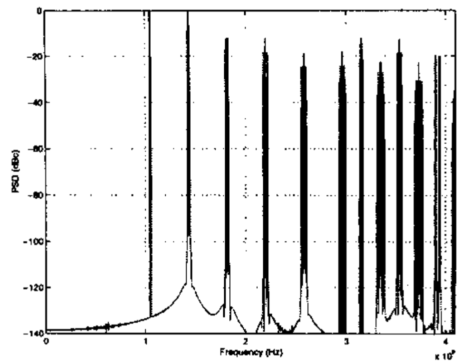

Figure 6. Far-out Spectrum of Output

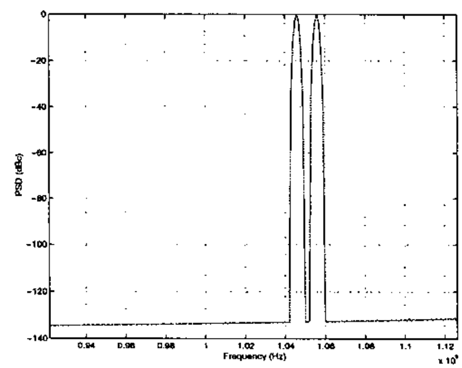

Figure 7. Near Spectrum of Output

5. DISCLAIMER

The technologies described in this paper are covered by a pending Motorola, Inc. patent.

6. REFERENCES

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