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Recursive Natural Sampling for Digital PWM

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ABSTRACT

This paper presents a highly accurate and computationally efficient method for digital-domain computation of naturally sampled digital pulse width modulation (PWM) signals. This method is used in a switching digital audio amplifier. The method is scalable for performance versus calculation complexity. Using a second order version of the algorithm with no iteration, intermodulation linearity of better than 113 dB is obtained with a full scale input at 19 kHz and 20 kHz. Matlab simulation and measured results from a digital amplifier implemented with this algorithm are presented. Overall system performance is not limited by the accuracy of the natural sampling method.

1. INTRODUCTION

Directly translating digital pulse code modulated (PCM) audio samples to a digital PWM duty ratio is called uniformly sampled PWM (UPWM). UPWM has a mathematical nonlinearity. Naturally sampled PWM (NPWM) is the signal which is determined by the intersection of a ramp and an analog signal. Natural sampling corrects for the mathematical nonlinearity of the PWM process. There exist numerous digital-domain methods for generating/approximating a naturally sampled digital PWM signal from a digital PCM input [1-3]. Some of these methods target digital audio amplifier applications [4-6] while others target inverter applications [7]. The method described in this paper achieves very high linearity with low computational overhead relative to previous work [8-9]. Further, the method is less sensitive to the presence of out-of-band noise in the incoming PCM signal. This method of natural sampling has been patented [10] but performance results have not been previously published.

2. IMPLEMENTATION

2.1. Description

Naturally sampled PWM is best described in the analog domain as comparing an analog input signal to a ramp waveform signal to produce a naturally sampled PWM output signal. In the digital domain one must interpolate between the incoming digital PCM audio samples to determine the cross point between a theoretical analog input and a theoretical ramp waveform. Figure 1 shows a block diagram of the recursive natural sampling algorithm. The input to the system is the uniform sampled input data. The output from the system is the naturally sampled output data. The duty ratio predictor block produces a guess time point (g) based on the uniform sample points and possibly previous natural sample points. The guess time point (g) and the uniform sample points (u) are used by the signal value interpolator block to produce an interpolated signal value (v). This signal (v) is passed to the duty ratio correction block. The guess time point (g) and the interpolated signal value (v) as well as the uniform sample points are used to correct for the error due to the difference of (g) and (v). In certain implementations the corrected value (x) is the natural sample output. In other implementations the corrected value (x) is the new guess and is passed to the signal value interpolator. The number of recursion steps depends on the accuracy and computation requirements.

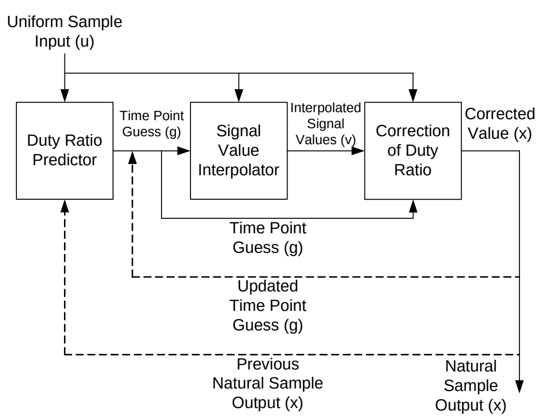

Figure 1 Signal flow of the natural sampling algorithm.

Figure 2 shows the natural sampling scheme in graphical form for a one-sided PWM system. In this figure, a theoretical ramp waveform having a normalized slope of one is shown along with a theoretical analog input signal that corresponds to the digital PCM input samples (u). The input to the process is the PCM input samples (u) which, for a one-sided PWM system, is provided at a sample rate equal to the PWM pulse repetition frequency. For each PWM pulse repetition period, an initial guess (g) is used for the desired cross point time of the theoretical analog input and ramp signals. Using the current and neighboring uniform PCM input samples, the value of the interpolated amplitude (v) at time point (g) is computed. The number of neighboring PCM input samples used for the calculation of the interpolated amplitude (v) equals the order of the system. Additionally, a simple estimate is made of the signal slope. The difference between the amplitude and time point (v-g) is used with the estimated signal slope to generate a correction (c) to produce a corrected value (x). Even with a poor guess, as in Figure 2, the difference between the corrected value (x) and the ideal value (i) is small. In practice, the uniform PCM input sample (u) can be used for the initial guess. The corrected value (x) can be used to be the guess for another round of interpolation, if greater accuracy is desired.

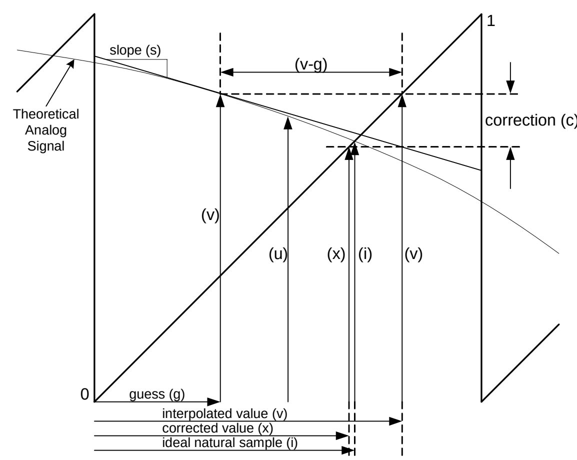

Figure 2 Signal diagram of the natural sampling process for one-sided PWM.

Figure 3 illustrates a two-sided PWM implementation. In this case, the ramp signal has equal rising and falling slopes. Consequently, the incoming PCM signal has a sample rate of twice the PWM pulse repetition frequency, and two natural sample points per pulse are calculated.

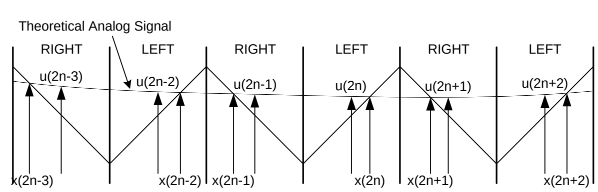

Figure 3 Signal diagram for two-sided PWM

2.2. Mathematical Derivation

The first step is determining a "guess" time point (g). In its most general form it is derived from a combination of previous, present and future uniform input sample points (u) as well as past-calculated natural samples. Example cases are as follows:

$$g(n) = u(n) + \left[2\{x(n-1) - u(n-1)\} - \{x(n-2) - u(n-2)\}\right] \quad (1)$$

or

$$g(n) = u(n) + \{x(n-1) - u(n-1)\} \quad (2)$$

or

$$g(n) = u(n) \quad (3)$$

The value of the signal at the guess time point is computed based on an interpolation formula. For two-sided PWM it is observed that better accuracy is achieved when the uniform samples are lined up with the center of the ramps, as shown in Figure 3. Using this approach, the interpolation of the signal value for the left sided PWM case can be calculated as follows:

$$\begin{aligned} v(2n) &= u(2n) + \left\{g(2n) - \frac{1}{2}\right\}\left\{\frac{u(2n+1) - u(2n-1)}{2}\right\} \\ &+ \left\{\frac{u(2n+1) + u(2n-1) - 2u(2n)}{2}\right\}\left\{g(2n) - \frac{1}{2}\right\}^2 \end{aligned} \quad (4)$$

For the right hand side a similar equation holds:

$$\begin{aligned} v(2n+1) &= u(2n+1) \\ &+ \left\{\frac{1}{2} - g(2n+1)\right\}\left\{\frac{u(2n+2) - u(2n)}{2}\right\} \\ &+ \left\{\frac{u(2n+2) + u(2n) - 2u(2n+1)}{2}\right\}\left\{\frac{1}{2} - g(2n+1)\right\}^2 \end{aligned} \quad (5)$$

Both of these equations are based on a three-point second order Lagrange interpolation formula. Other orders of Lagrange interpolation, as well as other types of interpolation formula, may be substituted based on accuracy and computational constraints.

The next step in the algorithm is the correction step. The correction step is based on the idea that if the guess is close to the ideal natural sample time point the interpolated value (v) is very close to the ramp. Thus, the signal value (v) must be very close to the time point guess (g). Any difference implies that the time point guess (g) is not entirely accurate and can be corrected assuming that the signal is slowly moving. A first order correction (c) for both the left side PWM and the right side PWM, respectively, is given below:

$$c(2n) = \{v(2n) - g(2n)\}\left\{\frac{u(2n+1) - u(2n-1)}{2}\right\} \quad (6)$$

$$c(2n+1) = \{v(2n+1) - g(2n+1)\}\left\{\frac{u(2n+2) - u(2n)}{2}\right\} \quad (7)$$

In both cases the corrected natural sample point output is determined by simply summing the interpolated signal value and the correction signal value.

$$x(m) = v(m) + c(m) \quad (8)$$

The correction term improves accuracy. Recursion can be introduced to further improve accuracy. Typically, one iteration improves accuracy significantly. Recursion can be implemented by using the corrected natural sample point output as a new time point guess as given by the equations below:

$$\begin{aligned} v_1(2n) &= u(2n) + \left\{g_1(2n) - \frac{1}{2}\right\}\left\{\frac{u(2n+1) - u(2n-1)}{2}\right\} \\ &+ \left\{\frac{u(2n+1) + u(2n-1) - 2u(2n)}{2}\right\}\left\{g_1(2n) - \frac{1}{2}\right\}^2 \end{aligned} \quad (9)$$

$$c_1(2n) = \{v_1(2n) - g_1(2n)\}\left\{\frac{u(2n+1) - u(2n-1)}{2}\right\} \quad (10)$$

$$g_2(2n) = x_1(2n) = v_1(2n) + c_1(2n) \quad (11)$$

The steps in equations (9)–(11) can be repeated for improved accuracy. The amount of recursion needed to achieve a given accuracy is dependent on the oversampling ratio. If the signal is moving fast and changing significantly from sample to sample, corresponding to a low oversampling ratio, the need for recursion increases. This comes with added computational and memory requirements. The complexity of each of the time point estimation, interpolation to compute signal value, and correction of signal value steps is variable depending on the accuracy required. Depending on the accuracy requirement, as well as constraints on memory and computation, an optimal algorithm can be designed. As an example, for a digital audio amplifier system with an input signal constrained to 20 kHz bandwidth and a 375 kHz PWM switching frequency using two-sided PWM the following left hand side equations can be used to achieve extremely good results:

$$g(2n) = u(2n) \tag{12}$$

$$v(2n) = u(2n) + \left\{u(2n) - \frac{1}{2}\right\}\left\{\frac{u(2n+1) - u(2n-1)}{2}\right\} + \left\{\frac{u(2n+1) + u(2n-1) - 2u(2n)}{2}\right\}\left\{u(2n) - \frac{1}{2}\right\}^2 \tag{13}$$

$$x(2n) = v(2n) + \{v(2n) - g(2n)\}\left\{\frac{u(2n+1) - u(2n-1)}{2}\right\} \tag{14}$$

The corresponding right hand side equations are as follows:

$$g(2n+1) = u(2n+1) \tag{15}$$

$$v(2n+1) = u(2n+1) + \left\{\frac{1}{2} - u(2n+1)\right\}\left\{\frac{u(2n+1) - u(2n-1)}{2}\right\} + \left\{\frac{u(2n+2) + u(2n) - 2u(2n+1)}{2}\right\}\left\{\frac{1}{2} - u(2n+1)\right\}^2 \tag{16}$$

$$x(2n+1) = v(2n+1) + \{v(2n+1) - g(2n+1)\}\left\{\frac{u(2n+2) - u(2n)}{2}\right\} \tag{17}$$

Note that in this example the initial time point guess (g) has been chosen to be the input uniform sample (u). Since the uniform sample (u) is already available there is no computation or memory storage associated with the guess. There are eleven multiply or add computations associated with the calculation of the interpolated signal value. There are four memory storage locations needed for this step. Calculation of the corrected natural sample point output requires an additional two add or multiply operations and uses one memory storage location. Thus the total is thirteen operations and five memory storage locations needed per sample. For a 375 kHz switching frequency with two samples per switching cycle, the total computation is 9.75 million operations per second. The low total memory requirement is particularly beneficial for reducing overall computation overhead.

3. PERFORMANCE

A two-sided PWM system was implemented in hardware. The hardware implementation incorporates three point second order Lagrange interpolation and uses the uniformly sampled PCM input signal as the initial guess. No additional iterations of the algorithm were used to calculate the naturally sampled output data. Figure 4 is a simulated spectrum of the two-sided natural sampling process with a 6.67 kHz input signal, showing a THD of -140 dB. Figure 5 shows the results of an intermodulation distortion (IMD) simulation consisting of a 19 kHz and 20 kHz dual tone input signal, showing an IMD of -113 dB. This compares to an IMD of -114.8 dB, as shown in Figure 6, when using an exact natural sample calculation of the interpolation formula utilizing the same number of PCM samples. This proves that the algorithm is close to optimal for a given order. Of course, if there is need for higher accuracy a higher order version of this algorithm may be used.

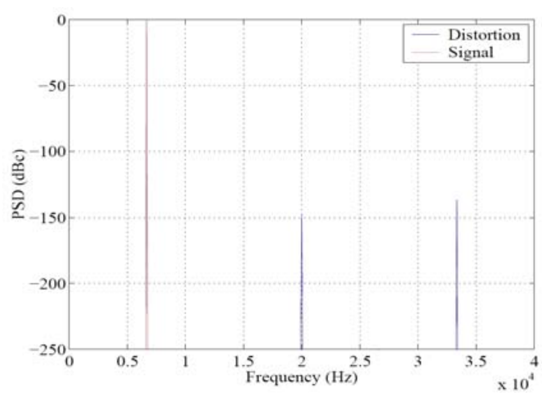

Figure 4 Simulated 6.67 KHz single tone spectrum of Natural Sampling (THD = -140dB)

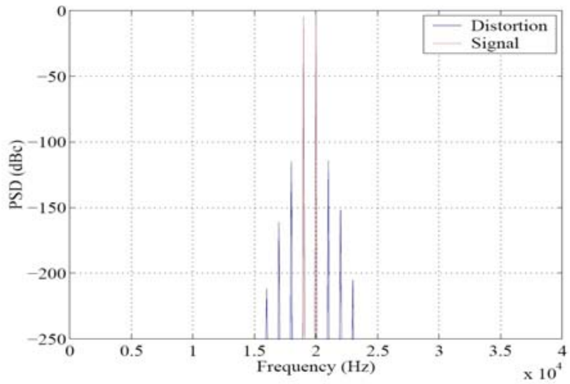

Figure 5 Simulated Two Tone Test (19 KHz and 20 KHz) using recursive natural sampling of PWM. In-band distortion = -113dB.

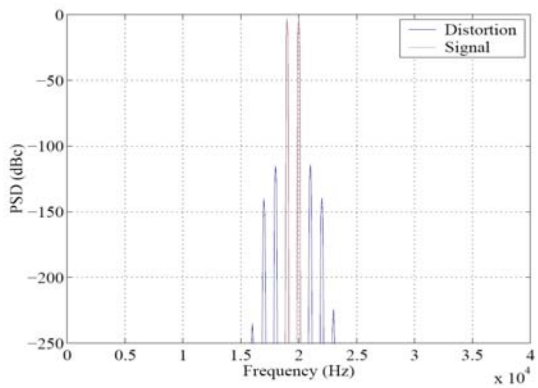

Figure 6 Simulated Two Tone Test (19 KHz and 20 KHz) using an exact natural sampling calculation with same-order interpolation. In-band distortion = -114.8dB.

This digital natural sampling algorithm has been implemented in a commercially available digital PWM controller IC (FSA95601). Figure 7 shows the simulated intermodulation distortion (IMD) spectrum and Figure 8 shows the measured spectrum with 19 kHz and 20 kHz tones at a -6 dB input level. The results are measured using Freescale Semiconductor's FSA95601 EVM board with discrete H-bridge power stages. In simulation, where the non-idealities of the power stage are not included, the intermodulation harmonics are buried in the noise floor. The measured spectrum, however, shows some nonlinearity, even with the use of a very high performance digital feedback loop. This shows that the nonlinearity shown in the measured spectrum is almost entirely due to the power stage.

The simulated and measured spectrums illustrate that the overall linearity of the system is not determined by the natural sampling algorithm. It also shows that using a higher order version of this natural sampling algorithm would not improve the overall signal fidelity for this application.

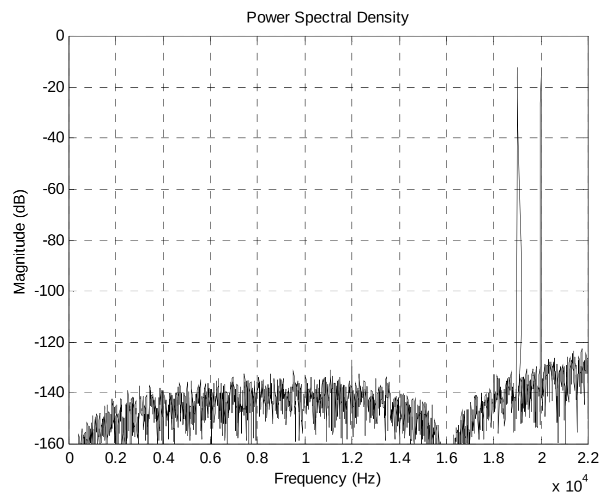

Figure 7 Simulated IMD, 19 kHz and 20 kHz input tones

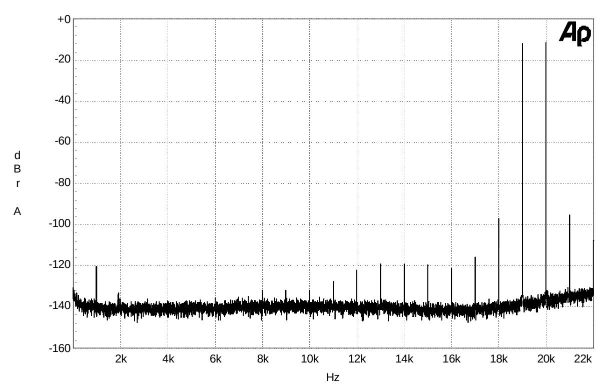

Figure 8 Measured -6dB IMD, 19 kHz and 20 kHz input tones

4. HIGH FREQUENCY CONTENT

High resolution audio using sample rates of 96 kHz and 192 kHz have gained in popularity. These audio signals can have spectral content at 40 kHz or higher. To investigate the effects of high frequency content on the natural sampling algorithm both IMD simulations and measurements were made using 29 kHz and 30 kHz dual tone inputs at -6 dB. Figure 9 shows the simulated result and Figure 10 shows the measured result. These results indicate that the natural sampling algorithm handles this higher frequency signal content with negligible in-band degradation due to intermodulation.

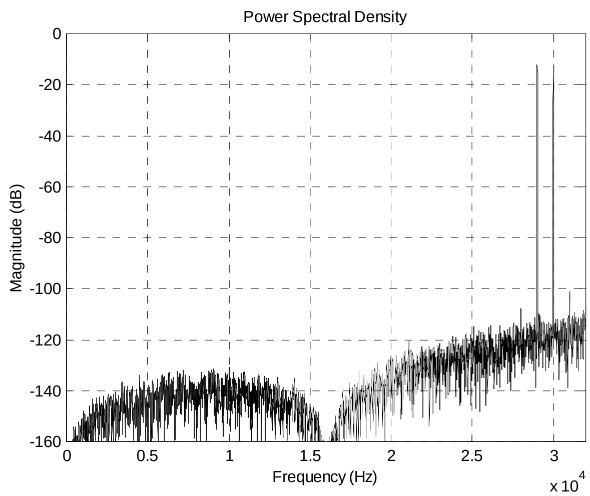

Figure 9 Simulated -6dB IMD, 29 kHz and 30 kHz input tones

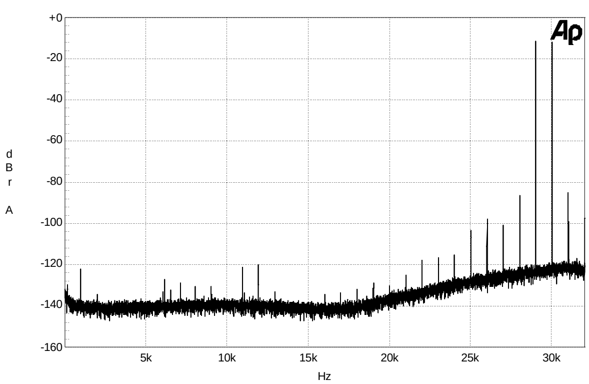

Figure 10 Measured -6dB IMD, 29 kHz and 30 kHz input tones

5. PCM OUT-OF-BAND NOISE

The digital output from a Super Audio Compact Disc (SACD) consists of a 1-bit pulse density modulated (PDM) bit stream sampled at 2.8224 MHz. A high order (typically seventh order) 1-bit sigma delta ADC is used to generate this bit stream, resulting in high resolution in the audio band and a large level of out-of-band noise. This digital PDM bit stream can be converted to a digital PWM signal by first decimating the sample rate from 2.8224 MHz to produce a high resolution digital PCM signal at either the PWM pulse repetition frequency (for one-sided PWM) or at twice the PWM pulse repetition frequency (for two-side PWM) [10]. This PCM signal can then be used as the input to a PCM-to-PWM conversion algorithm such as the one described in this paper.

A typical decimation process consists of a Comb filter followed by a FIR low pass filter. Achieving significant attenuation of the large out-of-band noise associated with the SACD signal requires a relatively expensive high-tap FIR low pass filter. This is illustrated in Figure 11 which shows the spectral content of the original SACD 1-bit PDM input signal, the output of a 4x decimating comb filter, and the output of both an ‘expensive’ 228-tap low pass filter and a ‘cheap’ 30-tap low pass filter.

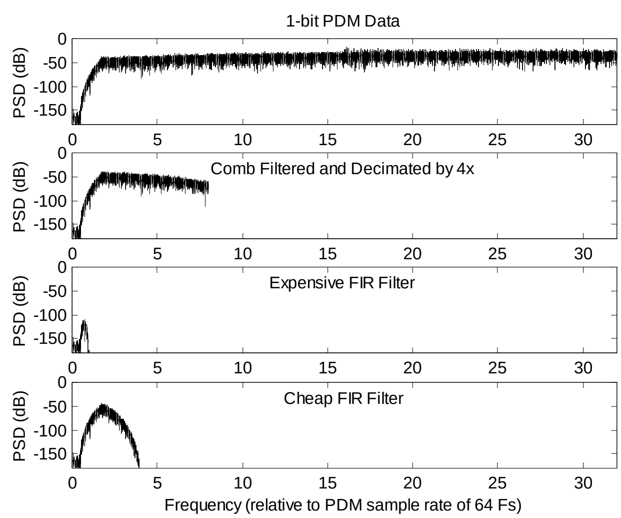

Figure 11 Spectral content of typical SACD 1-bit PDM data (1 kHz, 0dBFS input signal), subsequent comb filter output decimated by 4x, and comparison between an expensive 228-tap FIR low pass filter and a cheap 30-tap FIR low pass filter

The less expensive filter with fewer taps can be used if the PWM natural sampling algorithm is tolerant of the out-of-band noise. The recursive natural sampling algorithm presented in this paper is about 10 dB more tolerant of the out-of-band noise compared to a previous method using prediction filters [11]. This is illustrated in Figure 12 which compares the SNR of the PCM to PWM conversion algorithm for these two cases.

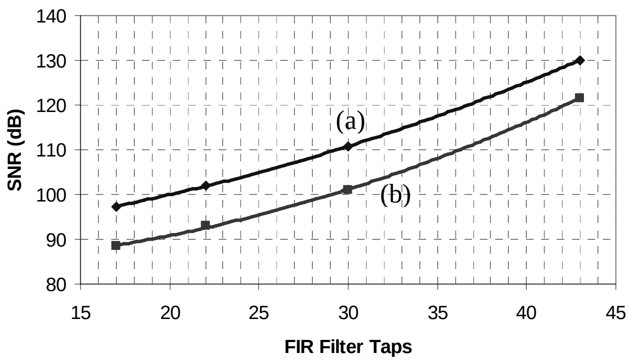

Figure 12 Affect of out-of-band noise in the PCM input signal on PWM output SNR for a) Recursive algorithm using no recursion (top), and b) alternate algorithm using prediction filtering (bottom)

6. CONCLUSION

This paper describes a highly accurate and efficient natural sampling algorithm for digital-domain conversion of PCM to PWM signals. Using only a handful of addition and multiplication operations the algorithm achieves a THD of -140 dB and an IMD of -113 dB measured over a 20 kHz bandwidth. The linearity is very close to the theoretical maximum for a curve fit algorithm using the same number of PCM samples. Additionally, the algorithm is relatively insensitive to both high frequency content and out-of-band noise in the PCM input signal. The overall performance at the output of the power stage is ultimately limited by the nonlinearities associated with the power stage.

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